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OPTIMUM WAVELET SELECTION FOR 2D SIGNAL USING SET PARTITIONING IN HIERARCHICAL TREES (SPIHT) METHOD

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Abstract: *Wavelet selection is most important part in signal compression as well as in signal denoising etc. In this paper we have proposed a technique of best wavelet selection for image processing using SPIHT method. By image compression, we can reduce the storage space of images that will be helpful to increase storage and transmission process's performance.* ***Compression is implemented in software using MATLAB Wavelet Toolbox [10], [11]. We have analysed two terms namely compression ratio and bit per pixel. Many wavelets are used in signal and image processing. We have taken Haar, Daubechies, Biorthogonal and Symlet wavelet out of these for best wavelet analysis.***

1. INTRODUCTION

Wavelet is a new development in the emerging field of data analysis for Physicists and Engineers [1, 2]. It represents an efficient computational algorithm under the interest of a broad community. In wavelet we use a single function and its dilation and translation to generate a set of orthonormal basis functions to represent a signal. Number of such functions is infinite and we choose one that suits to our application. The range of interval over which scaling function and wavelet function are defined is known as support of wavelet [3]. Beyond this interval (support) the functions should be identically zero. Wavelet theory has recently been used to trace the changes in trends and to detect non-stationarities (interruptions or non-persistence in the temperature variability record) is particularly crucial when making statistical interferences. These earlier in-homogeneity analysis models filtered out inhomogeneous as differences compared to specific trends (regressive trend) or bandwidths (by using low pass filters utilizing moving averages), while wavelet transform presents in-homogeneity in time series as the sum of temporal changes in the amplitude and phase of records over a wide since wave bandwidth. Spectral analysis of river Ramganga hydraulics and biochemical oxygen demand in river water using discrete wavelet transforms are studied by Kumar et al. [4, 5]). We discuss some wavelets in brief as following:

1.1 Haar wavelet

Haar wavelet is the simplest possible wavelet. It is not continuous and therefore not differentiable. This is an advantage for the analysis of signals with sudden transitions. It conserves the energy of signal and compaction of the energy of signals. In discrete form, Haar wavelets are related to a mathematical operation called the Haar transform. The Haar transform has been mainly used for image processing and pattern recognition due to its low computing requirements. The two dimensional signal processing is an area of efficient applications of Haar transform due to their wavelet like structure. It is found effective in applications such as signal and image compression in electrical and computer engineering as it provides a simple and computationally efficient approach for analysing the local aspects of a signal. It shows orthogonal, biorthogonal and compact support.

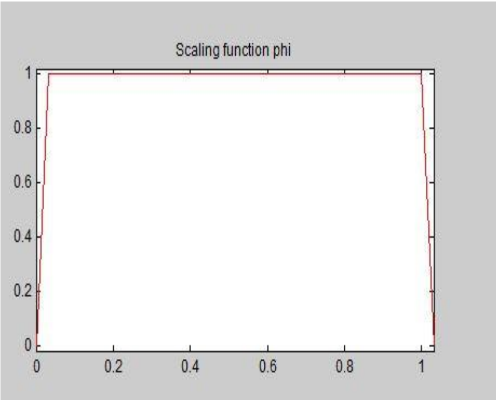

Fig.1.1(a): Scaling function of Haar

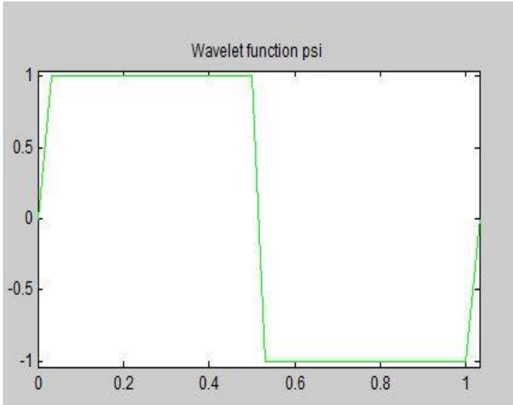

Fig. 1.1(b): Wavelet function of Haar

1.2 Biorthogonal wavelet

This family of wavelets exhibits the property of linear phase, which is needed for signal and image reconstruction [6]. Biorthogonal Wavelets are families of compactly supported symmetric wavelets. The symmetry of the filter coefficients is often desirable since it results in linear phase of the transfer function. In the Biorthogonal case, rather than having one scaling and wavelet function, there are two scaling functions ϕ and ϕ^* that may generate different multiresolution analysis, and accordingly two different wavelet functions ψ and ψ *. ψ^* is used in the analysis and ψ is used in the synthesis.

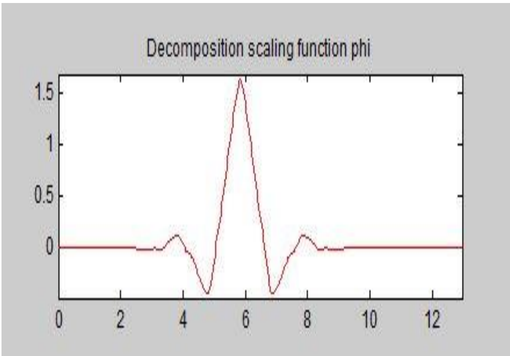

Fig.1.2(a): Scaling function of Biorthogonal

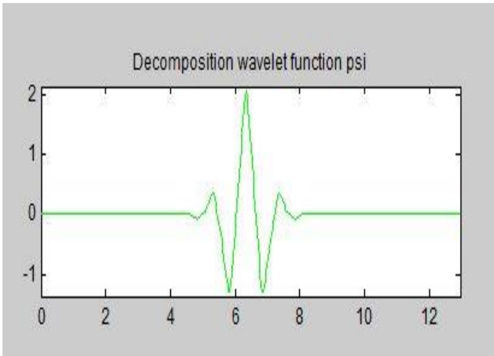

Fig.1.2(b): Wavelet function of Biorthogonal

1.3 Daubechies wavelet

It is a family of wavelet transform discovered by Ingrid Daubechies [7]. Its concept is similar to Haar but differs in scaling and wavelet functions. It defines a DWT and characterized by a maximal number of vanishing moment for some given supports. A vanishing moment limits the wavelets ability to represent polynomial behaviour or information in a signal.

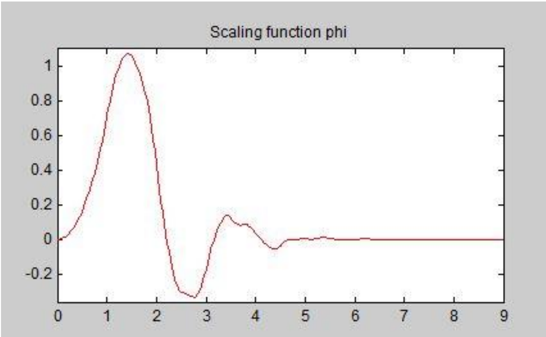

Fig.1.3(a): Scaling function of Daubechies

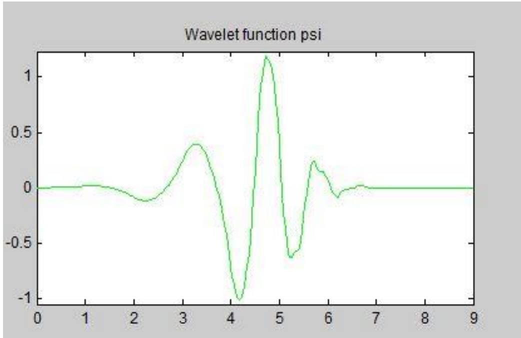

Fig.1.3(b): Wavelet function of Daubechies

1.4 Symlet Wavelet

The Symlet wavelet is also known as Daubechies least symmetric wavelets [8]. The Symlets are more symmetric than external phase wavelets. In SymN, N is the number of vanishing moments. It shows orthogonal, biorthogonal and compact support.

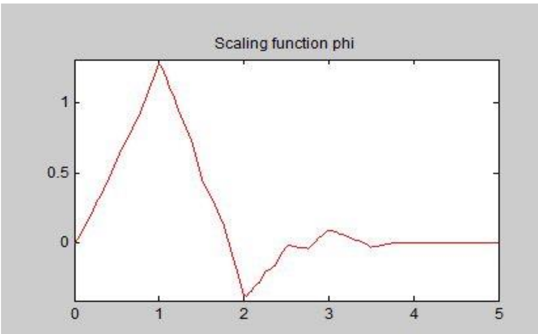

Fig1.4(a): Scaling function of Symlet

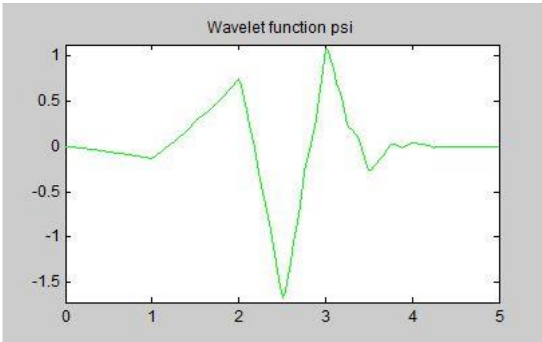

Fig1.4(b): Wavelet function of Symlet

2. MULTIRESOLUTION ANALYSIS

Wavelets are a special kind of functions which exhibit oscillatory behaviour for a short period of time and then die out. For any two real numbers *a* and *b,* a wavelet function is defined as:

$$\psi_{a,b} = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)$$

If we choose $a=2^{-j}$ and $b/a = k$, we get discrete wavelets as following:-

$$\psi_{j,k}(t) = 2^{j/2}\psi(2^{j}t - k)$$

The wavelet transform of a signal captures the localized time frequency information of the signal. A multi resolution analysis (MRA) [6] is a radically new recursive method for performing discrete wavelet analysis. A MRA for introduced by Mallat [2, 3] and extended by other researchers consists of a Sequence V_j j ∈ Z of closed subspaces of $L^2(\mathbb{R})$, a space of square integrable functions, satisfying the following properties;

1) $V_{j+1} \subset V_j$: $j \in \mathbb{Z}$

2) $\cap_{j\in Z} V_j=\{0\}$, $\cup_{j\in Z}=$ $L^2(\mathbb{R})$,=$\{0\}$, $\cup_{j\in Z}=$ $L^2(\mathbb{R})$,

3) For every, $L^2(\mathbb{R})$, f (t) ∈ $V_j \Rightarrow f\left(\frac{t}{2}\right) \in V_{j+1}$, $\forall\ j \in \mathbb{Z}$

4) There exists a function $\phi(t) \in V_o$ such that $\{\phi(t-k): k\in\mathbb{Z}\}$ is orthonormal basis of V_o.

The function ϕ (t) is called scaling function of given MRA and property 3 implies a dilation equation as following.

$$\phi(t) = \sum_{k\in Z} h_k\ \phi(2t-k)$$

Where h_k is low pass filter and is defined as:

$$h_k=\left(\frac{1}{\sqrt{2}}\right)\int_{-\infty}^{\infty}\phi(\mathrm{t})\phi(2\mathrm{t}-\mathrm{k})\,dt$$

Now we consider W_0 be orthogonal compliment of V_0 in V_1 i.e.

$$V_1= V_0 \oplus W_0$$

If $\psi \in W_o$ be any function then,

$$\psi(t) = \sum_{k\in Z} h_k\ \phi(2t-k)$$

where, $g_k=(-1)^{k+1}\ h_{1-k}$ are high pass filters.

We can express a signal in terms of bases of V_0 space and W_0 space. If we combine the bases of V_0 and W_0 space, we can express any signal in V_0space. Using the same argument, we can write

$$V_2=V_1 \oplus W_1$$

In general,

$$V_j = V_{j-1} \oplus W_{j-1}$$

But, $$V_{j-1} = V_{j-2} \oplus W_{j-2}$$

Therefore

$$V_j = W_{j-1} \oplus W_{j-2} \oplus V_{j-2}$$

.. ..

$$V_j = W_{j-1} \oplus W_{j-2} \oplus W_{j-3} \oplus \ldots\ldots\ldots\ldots \oplus W_0 \oplus V_0$$

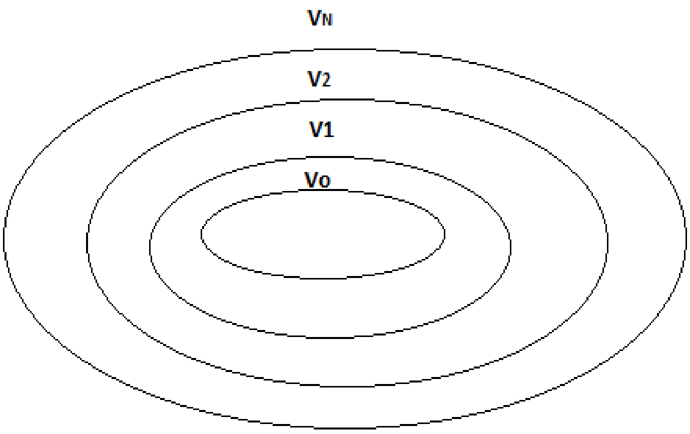

Fig. 2.1: Signal spaces

Let S= $\{S_n$: n∈$\mathbb{Z}\}$ be a function sampled at regular time interval, $\Delta t = \tau$ where $\mathbb{Z}$ is an integer. S is split into a "blurred" version a_1 at the coarser interval$\Delta t = 2\tau$ and "detail" d_1 at scale $\Delta t = \tau$.This process is repeated and gives a sequence S, a_1 , a_2, a_3, a_4 ....of more and more blurred versions together with the details d, d_2, d_3, d_4, …….removed at every scale (Δt =$2^m\tau$in a_m and d_{m-1}). Here a_{m}'s and d_{m}'s are approximation and details of original signal. After N iteration S can be reconstructed as S = a_N + d_1 + d_2 + d_3 + d_4 + d_5 + ...+ d_N. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components. Thus the original signal, S, passes through two complementary filters in which one is low pass filter and second one is high pass filter as shown in figure below.

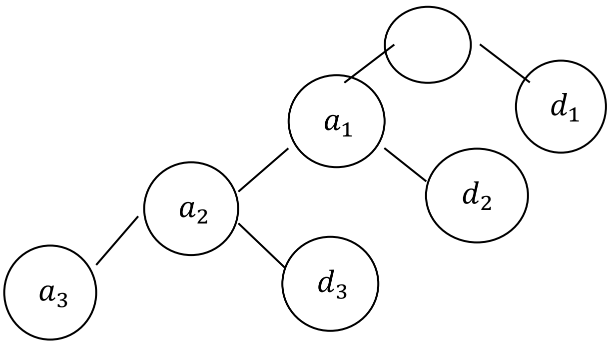

Fig. 2.2: Signal decomposition

where

$$S = a_1 + d_1$$
$$= a_2 + d_2 + d_1$$
$$= a_3 + d_3 + d_2 + d_1$$
$$= a_4 + d_4 + d_3 + d_2 + d_1$$

and so on.

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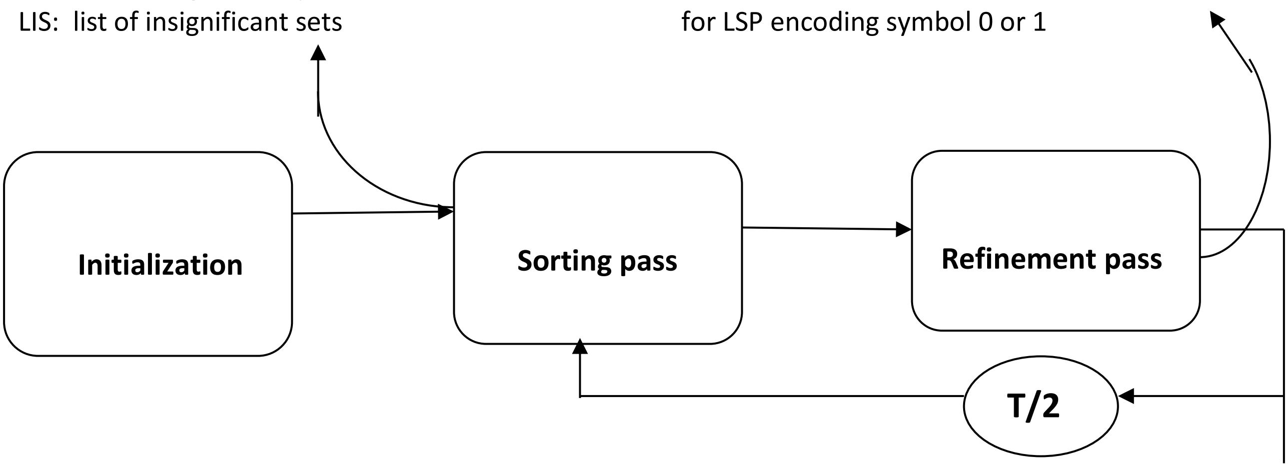
3. SPIHT ALGORITHM SET IN HIERARCHICAL TREES

The SPIHT method is not a simple extension of traditional method for image compression but it represents an important advanced in the field. It is one of the best algorithm in term of Peak- Signal-to- noise ratio and execution time. This method provides good image quality, high PSNR, especially for color images. SPIHT [9] utilies the properties of the wavelet transformed images to increase its efficiency. It is optimized for progressive image transmission and also produces a fully embedded coded file. It can be used for lossless compression.We can code to exact bit rate or distortion. The images obtained with wavelet- based methods yield very good results when combined with wavelets. SPIHT defines parent children relationships between these self similar sub bands to establish spatial orientation trees. The image is first decomposed into a series of wavelet coefficient. These coefficients are then grouped into sets knowns as spatial Orientation tree. After that, the coefficient in each spatial orientation tree are encoded progressively from the most significant bit planes, starting with coefficient with highest magnitude.

The SPIHT algorithm involves two coding passes: the sorting pass and the refinement pass as shown in figure 3.1. The sorting pass looks for zero trees and sorts significant and insignificant coefficients with respect to a given threshold. And the refinement pass sends the precision bits of the significant coefficients. After one sorting pass and one refinement pass, which can be considered as one scan pass, the threshold is halved, and the coding process is repeated until the expected bit rate is achieved.

3.1 SPIHT Algorithm

LSP: list of significant pixels
LIP: list of insignificant pixels
LIS: list of insignificant sets



T: Threshold

Fig.3.1: SPIHT Algorithm

3.2 Performance Parameter

Compression is achieved by different copmprission performance parameter such as BPP(bit per pixel) ratio, L2 norm ratio, compression ratio(CR).

Compression Ratio: It is te ratio of amount of bit in the original image to the amount of bit in compressed image.It is the reconstruction performance of compressed image in term of storage capacity.

L2 norm ratio: It is the confirmity term which ensures precision of preliminary course of estimation.

Bit per pixel (BPP): It represent the number of bits required to store one pixel of image. Bit rate is an average bits required to represent a single sample (pixel) in the compressed image. The standard value of the bit rate for image is 8 bits per pixel (bpp). The initial BPP is 8 for a greyscale image. The initial BPP is 24 for a true colour image, because 8 bits are used to encode each of the three colours (RGB colour space). The value of bit rate will lie in between 0 and 8.

4. EXPERIMENTAL ANALYSIS

We select original image as in figure 2 for experiment analysis. We take all wavelets of Daubechies, Biorthogonal, Haar and Symlet families for all levels. We have selected SPIHT method and take encoding loop 12. We finally reach on the conclusion that Daubechies 5, Biorthogonal 2.6, Haar and Symlet 3 gives better result for level 4. We select wavelets given in table 1 and table 2 on the basis of PSNR and MSE [10].

Fig. 4.1: Original image

In table 1, we discuss the wavelet db 5, Bior 2.6, sym 3 and Haar on the basis of bit per pixel, L2 norm ratio, compression ratio and compressed size of image. Similarly, in table 2, we repeat above process for level 8.

Level -4

Wavelet Name	BPP	L2 Norm ratio	Compression Ratio	Compressed Size in KB
Daubechies 5	7.905	99.97%	33.98%	127
Biorthogonal 2.6	7.643	99.97%	31.84%	129
Haar	8.143	100.01%	33.93%	107
Symlet 3	8.136	99.97%	33.90%	124

(Table 1)

Level 8

Wavelet Name	BPP	L2 Norm ratio	Compression Ratio	Compressed Size in KB
Daubechies 5	0.905	99.76%	3.77%	114
Biorthogonal 2.6	1.024	99.83%	4.25%	105
Haar	1.080	99.77%	4.50%	23.8
Symlet 3	0.909	99.78%	3.79%	112

(**Table 2)**

5. RESULT AND DISCUSSSION

From table 1 and table 2, it is obvious that Haar wavelet gives better results because it reduces the size of image by 23.8 KB for level 4 and 107 KB for level 8, so that the compression ratio is 33.93% for level 4 and 4.50% for level 8 [11, 12]. Compression ratio is greater for both level as compared to that of other wavelets. We know that greater the compression ratio gives better compression. L2 norm ratio is 100.01% for level 4 and 99.97% for level 8 which give better precision quality which can see in figure 5.1 and figure 5.2. Normalized histogram, compressed image, original decomposition, compressed image decomposition of Haar wavelet for level 4 and for level 8 are also shown in fig. 5.1 and fig. 5.2.

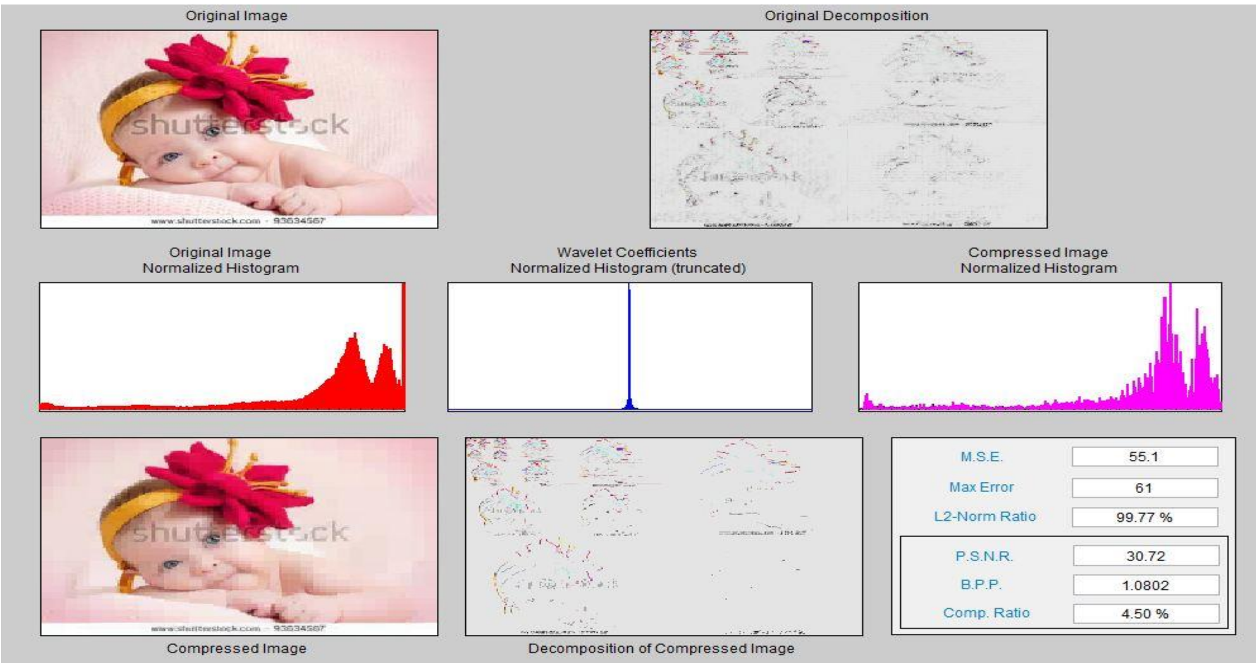


Fig 5.1: Image compression by Haar wavelet level 4

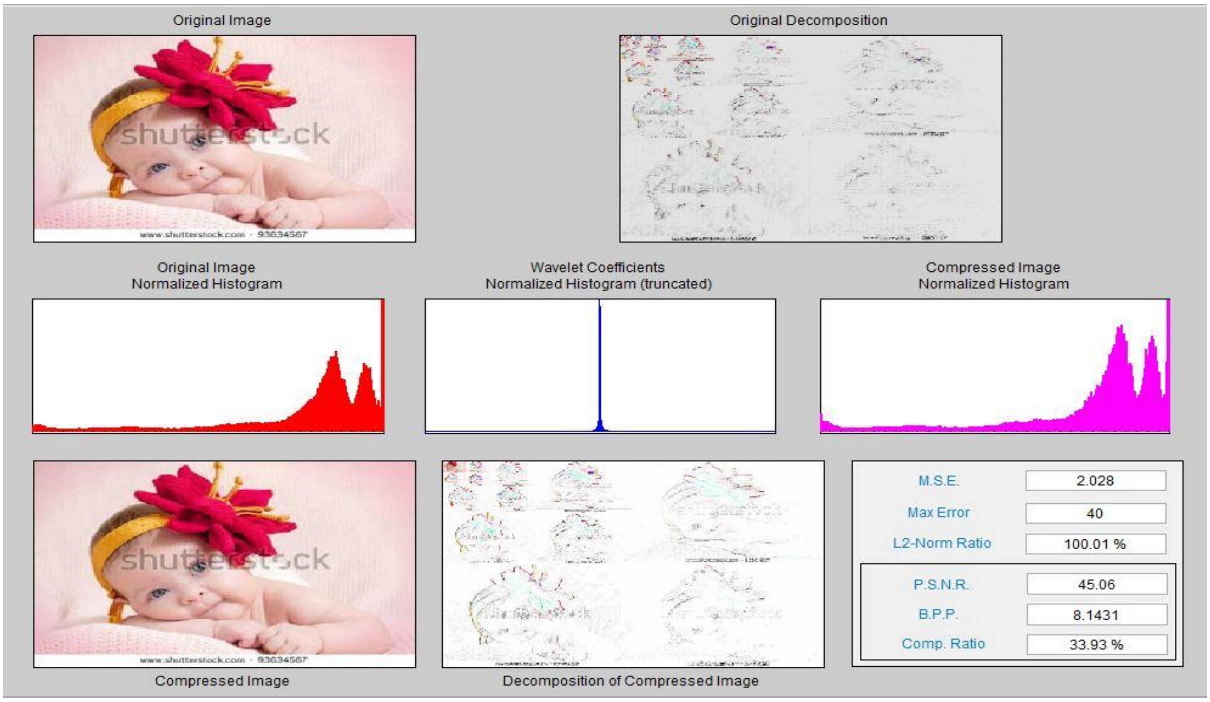


Fig. 5.2: Image compression by Haar wavelet level 8

6. CONCLUSION

The wavelet transforms using multiresolution analysis perform averaging and differencing between pixel values to form the approximation and detail coefficients of a signal (image). All the detail coefficients are zero, which means that 100% of the energy can be compacted into the approximation sub signal giving better compression. The other wavelets used are however not as good for this image, they try to find more sudden changes in the image and look at how the intensity values are changing over a greater number of pixels. Analytical study of image compression using Haar wavelet and following SPIHT method, it is obvious that Haar wavelet is the optimum and best wavelet for the compression of considered image.

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